THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2010D Advanced Calculus 2019-2020

Problem Set 2

1. Let $A = (0, 2, 3, 3)$ and $B = (1, -1, 2, 3)$ be two points in $\mathbb{R}^4$. Find the equation of straight line passing through A and B express it in standard form.

2. Find the equation of the plane Π containing the straight line
$$L : \frac{x-4}{2} = \frac{y-3}{5} = \frac{z+1}{-2}$$
and the point $P(2, -4, 2)$.

3. Find the equation of the straight line given by the intersection of two planes $\Pi_1 : x + y - z = 1$ and $\Pi_2 : x + 2y + 2z = 3$.

4. Let $\Pi : x_1 + 3x_2 - 2x_3 + x_4 + 3 = 0$ be an affine hyperplane and let $P = (7, 21, -7, 3)$ be a point in $\mathbb{R}^4$.
   (a) Find the projection Q of the point P on Π.
   (b) Find the image P' of P under the reflection across Π
   (c) Let $L : (x_1, x_2, x_3, x_4) = (7, 21, -7, 3) + t(3, 10, -4, 4)$ for $t \in \mathbb{R}$, be a straight line passing though P. Find the equation of the straight line L' which is the reflection of L across Π.

5. Find the equation(s) of the plane(s) Π such that Π is parallel to the plane $\Pi' : x + 2y - 2y + 3 = 0$ and the distance between the origin and Π is 4 units.

6. Let $L_1 : \frac{x+2}{3} = \frac{y-3}{4} = z - 2$ and $L_2 : x - 3 = 5 - y = 1 - z$ be two straight lines in $\mathbb{R}^3$.
   (a) Prove that L_1 and L_2 intersect at a point and find the coordinates of that point.
   (b) Find the acute angle between L_1 and L_2.
   (c) Find the equation of the plane containing L_1 and L_2.

7. Let Π be an affine hyperplane in $\mathbb{R}^n$ given by the equation $A_1x_1 + A_2x_2 + \cdots + A_nx_n + B = 0$ and let $P(p_1, p_2, \ldots, p_n)$ be a fixed point.
   Show that the perpendicular distance between Π and P is $\left| \frac{A_1p_1 + A_2p_2 + \cdots + A_np_n + B}{\sqrt{A_1^2 + A_2^2 + \cdots + A_n^2}} \right|$.

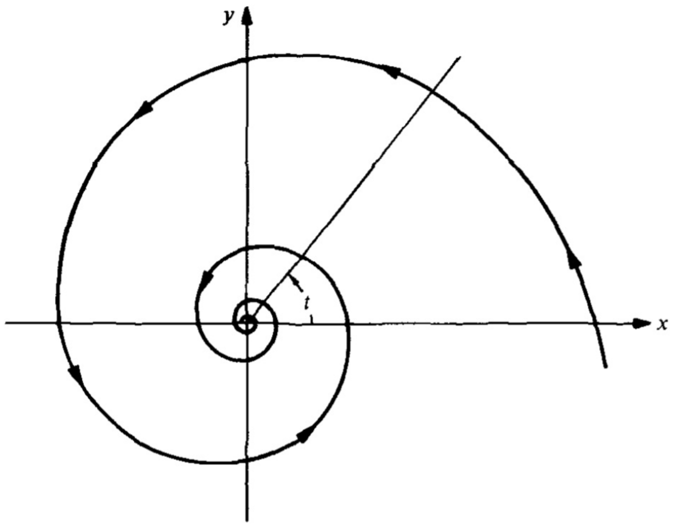
8. Suppose that $\Pi_1 : x + y + z = 1$ and $\Pi_2 : x - y + z = 2$ are two planes in $\mathbb{R}^3$.
   (a) Show that the intersection of Π_1 and Π_2 is a straight line and find a parametric equation of that line.
   (b) Find the equation(s) of the plane(s) containing all the points which are equidistant from Π_1 and Π_2.

9. Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a curve defined by $\gamma(t) = (\cos 2t - 1, \sin 2t + 2)$.
   (a) Write down an equation of γ in x and y only. What is γ?
   (b) Find $\gamma'(t)$.

10. Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a curve defined by $\gamma(t) = (4 \cos 2t, 9 \sin 2t)$.
    (a) Write down an equation of γ in x and y only. What is γ?
    (b) Find $\gamma'(t)$.

11. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$. Parametrize the straight line γ which passes through $\mathbf{a}$ and $\mathbf{b}$.

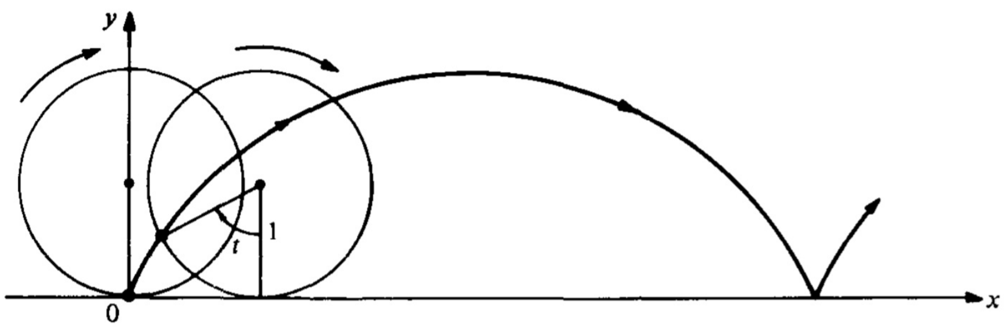
12. Let $\gamma(t) = (ae^{-bt}\cos t, ae^{-bt}\sin t)$ for $t \in \mathbb{R}$, where $a, b > 0$, which is called the *logarithmic spiral*.



(a) Show that as $t \to +\infty$, $\gamma(t)$ approaches the origin.

(b) Show that $\lim_{t\to+\infty} \int_0^t |\gamma'(t)|\, dt$ is finite, that is γ has finite arc length in $[0, +\infty)$.

13. In the following diagram, a circular disk of radius 1 in the plane xy rolls without slipping along the x-axis and the curve is the locus of a fixed point on the circumference which is called a *cycloid*.



(a) Give a parametrization of the cycloid.

(b) Find the arc length of the cycloid corresponding to a complete rotation of the disk.